



**CONTRIBUTION TO THE RESEARCH OF SIDESLIP ANGLE  
INFLUENCE ON VEHICLE STEERABILITY**

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RESEARCH ARTICLE

**ABSTRACT:** Knowing the limits of transverse handling and vehicle stability is very important from the aspect of designing a system aimed at improving handling and safety. One of the phenomena that have a great impact on vehicle stability and handling is the sideslip angle. Unfortunately, direct measurement of the sideslip angle requires very complex and expensive equipment that is not suitable for installation on ordinary passenger vehicles; therefore, this quantity must be estimated on the basis of measurements of other parameters. As a result of limited information, this parameter is mainly estimated or linearized relative to lateral force when used for calculations though such calculations can be less accurate. In this paper it is concluded that a clear relationship can not be formed between the lateral force and the sideslip angle and that for precise determination of the sideslip angle several parameters need to be known such as tire construction, tire pressure, rim diameter, wheel camber as well as micro and macro road structure.

**KEY WORDS:** Sideslip angle, steerability, lateral force

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## **PRILOG ISTRAŽIVANJU UTICAJA UGLA POVOĐENJA TOČKOVA NA UPRAVLJIVOST VOZILA**

**REZIME:** Poznavanje granica poprečnog upravljanja i stabilnosti vozila veoma je važno sa aspekta projektovanja sistema usmerenog na poboljšanje upravljanja i bezbednosti. Jedan od fenomena koji ima veliki uticaj na stabilnost i upravljivost vozila je povodjenje točkova. Nažalost, direktno merenje ugla povodjenja točkova zahteva vrlo složenu i skupu opremu koja nije pogodna za ugradnju na obična putnička vozila; zbog toga se ova veličina mora proceniti na osnovu merenja drugih parametara. Kao rezultat ograničenih informacija, kao i činjenice da se bočni koeficijent prijanjanja i ugao povodjenja brzo menjaju u zavisnosti od vrste i kvaliteta puta ili prilikom preduzimanja određenih manevara, ovaj parametar se uglavnom procenjuje ili se vrši njegova linearizacija u odnosu na bočnu silu kada se koristi za proračune, mada takvi proračuni mogu biti manje tačni. U ovom radu se zaključuje da se ne može stvoriti jasan odnos između bočne sile i ugla povodjenja točkova i da je za precizno određivanje ugla povodjenja potrebno znati nekoliko parametara kao što su konstrukcija pneumatika, pritisak u pneumaticima, prečnik oboda, ugao nagiba točka kao i mikro i makro reljef puta.....

**KLJUČNE REČI:** ugao povodjenja točkova, upravljivost, bočna sila

# CONTRIBUTION TO THE RESEARCH OF THE SIDESLIP ANGLE INFLUENCE ON VEHICLE STEERABILITY

*Stefan Milićević*

## INTRODUCTION

Knowing the limits of transverse handling and vehicle stability is very important from the aspect of designing a system aimed at improving handling and safety. Such systems tend to prevent unwanted vehicle behavior by using active control systems and assisting the driver in controlling the motor vehicle. Examples of such systems are the Anti-lock braking system (*ABS*), traction control system (*TCS*), and Electronic Stability Program (*ESP*) [2].

One of the phenomena that have a great impact on vehicle stability and handling is the sideslip angle. When a high-intensity lateral force acts on the vehicle during movement, due to their elasticity, tire deformation occurs, and, as a result, the direction of vehicle movement changes. This problem is especially present when turning, when, due to centrifugal or some other force (e.g. wind force), oversteer or understeer occur. In many cases, the driver is unable to correct the vehicle's trajectory and accidents can occur. Therefore, knowledge of the vehicle's lateral performance is very important because it directly affects the occurrence of oversteer and understeer, and the main factor is the phenomenon of the sideslip angle.

Knowledge of the vehicle's sideslip angle, which relates its lateral velocity to its longitudinal velocity, is largely unavailable for current safety systems; systems such as ESC have access to measurements of sideslip rate, not sideslip angle [2]. Unfortunately, direct measurement of the sideslip angle requires very complex and expensive equipment that is not suitable for installation on ordinary passenger vehicles; therefore, this quantity must be estimated on the basis of measurements of other parameters such as lateral and longitudinal acceleration, velocity, yaw rate, and steer angle. The difficulty in slip angle estimation is due to nonlinear characteristics of tires and influence of relative slant of the road surface. Methods that design observers to estimate sideslip often depend on accurate tire parametrization, which is problematic since tire parameters vary based on the road surface [3-5].

As a result of limited information, as well as the fact that the lateral coefficient of adhesion and sideslip angle change rapidly depending on the type and quality of the road soil or when undertaking certain emergency maneuvers, this parameter is mainly estimated or linearized relative to lateral force (Figure 1.c) when used for calculations though such calculations can be less accurate.

However, if a clear relationship was to be formed between the lateral force and the sideslip angle, at least for the adopted road parameters and the shape and construction of the tires, it would be possible to determine the sideslip angle at every moment. In the case where it is possible to determine the sideslip angle, it is also possible to design a system where the handling control, e.g. via steer-by-wire, would be near perfect..

## 1 BASIC CONSIDERATIONS ON SIDESLIP ANGLE AND VEHICLE STEERABILITY TESTING

The motion of a vehicle is governed by the forces generated between the tire and the road. In this part of the paper, an important concept related to the characterization of forces between tires and road is described. It is explained how lateral tire force is generated as a function of tire deformation.

Due to the elasticity of the tire, a lateral deformation is formed under the action of a force ( $F_y$ ) that acts in the direction normal to the tire rolling plane (Figure 1).

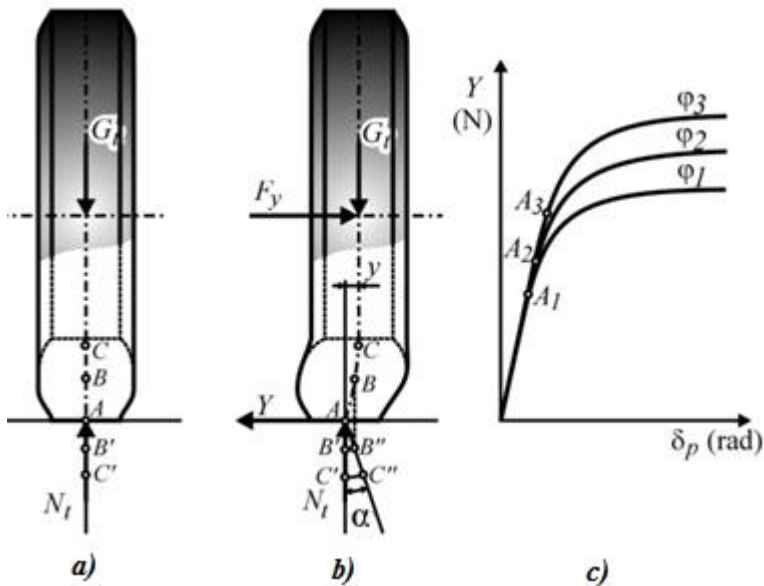


Figure 1. Front view of a laterally deflected tire

The force causes a horizontal reaction of the road ( $Y$ ) which is parallel to it and a linear displacement ( $y$ ) of the force ( $G_t$ ) relative to the contact surface and the normal reaction ( $N_t$ ). If points A, B, and C are observed on the circumference of the tire symmetrical ridge, it is obvious that they will come into contact with the road at points A', B' and C' found in the rolling plane when rolling without lateral force (Figure 1.a), ie. the tire trajectory is a line located in rolling plane. If, however, there is a lateral force ( $F_y$ ) and a linear displacement ( $y$ ), points A, B and C no longer lie in the same plane, so when rolling the wheel will come into contact with the ground at points A'', B'' and C'' (Figure 2.b), ie. the tire trajectory is a line which is at an angle  $\alpha$  relative to the rolling plane. The phenomenon of a change in the angle of the wheel trajectory relative to the rolling plane is called the sideslip angle.

Lateral force appears as a consequence of external influence, e.g. strong crosswind, or as a consequence of turning, ie. consequence of centrifugal force.

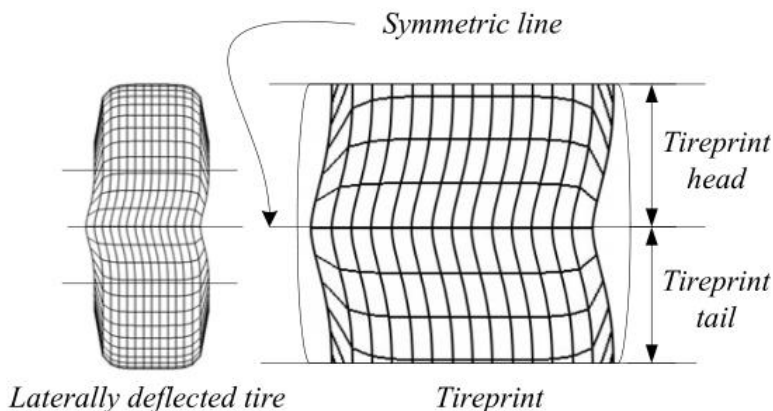


Figure 2. Bottom view of a laterally deflected tire, [7]

In the literature [6,7,8] the relation is considered to be as follows:

$$Y = K_p \cdot \alpha, \tag{1}$$

where:  $K_p$  - cornering stiffness of the tire.

The coefficient  $K_p$  is defined [7]:

$$K_p = \lim_{\alpha \rightarrow 0} \frac{\partial(-Y)}{\partial \alpha} = \left| \lim_{\alpha \rightarrow 0} \frac{\partial Y}{\partial \alpha} \right|, \tag{2}$$

A sample of measured lateral force  $F_y$  as a function of slip angle  $\alpha$  for a constant vertical load is plotted in Figure 3. [7]. The lateral force  $F_y$  is linear for small slip angles, however the rate of increasing  $F_y$  decreases for higher  $\alpha$ . The lateral force remains constant or drops slightly when  $\alpha$  reaches a critical value at which the tire slides on the road. Therefore, we may assume the lateral force  $F_y$  is proportional to the slip angle  $\alpha$  for low values of  $\alpha$  as described with the above equations [7].

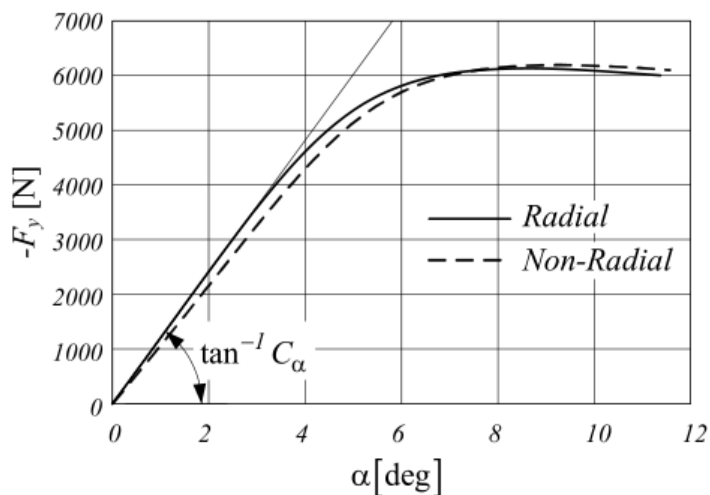


Figure 3. Lateral force  $F_y$  as a function of slip angle  $\alpha$  for a constant vertical load, [7]

However, to determine the true value of the sideslip angle, the researcher needs the lateral forces (or the coefficient of friction) based on the slip angle and the parameters:

vertical force (or wheel load) in the centre of tire contact

- tire pressure
- wheel camber
- tire type.

The influence of the sideslip angle is most obvious in the behavior of the vehicle during turning. Due to the elasticity of the tires, there is a deformation of the part of the tire in contact with the road and the difference between the velocity vector of the vehicle and the rolling plane of the wheels. A similar thing happens on the rear wheels of vehicles with the sideslip angle of the opposite sign (Figure 4).

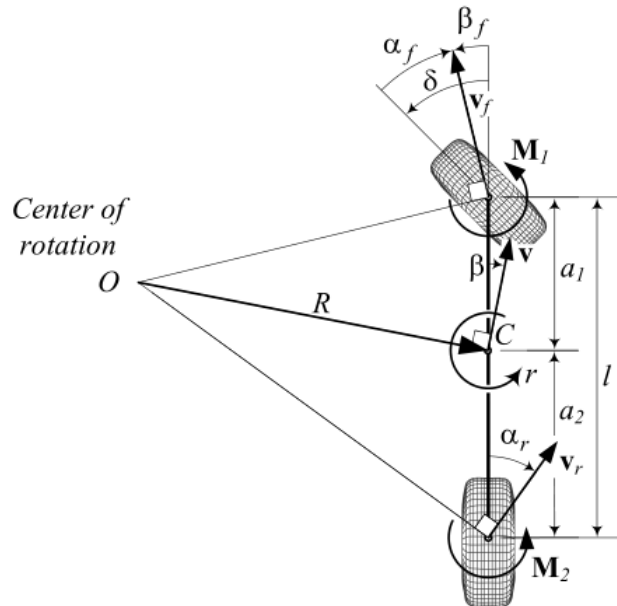


Figure 4. Cornering of a bicycle model, [7]

If its adopted:

$$K = W_f / K_f - W_r / K_r, \quad (3)$$

where:

- $W_f$  - load on the front axle
- $W_r$  - load on the rear axle
- $K_f$  - cornering stiffness of the front tires
- $K_r$  - cornering stiffness of the rear tires

and to be called "understeer gradient" then depending on the size of the steering angles on the front and rear wheels there are three cases [11]:

Neutral Steer -  $\alpha_f = \alpha_r$ ,  $K = 0$

On a constant-radius turn, no change in steer angle will be required as the speed is varied. Specifically, the steer angle required to make the turn will be equivalent to the Ackerman

Angle, 57.3 L/R. Physically, the neutral steer case corresponds to a balance on the vehicle such that the force on the lateral acceleration at the center of gravity causes an identical increase in slip angle at both the front and rear wheels.

Understeer -  $\alpha_f > \alpha_r$ ,  $K > 0$

On a constant-radius turn, the steer angle will have to increase with speed in proportion to  $K$  times the lateral acceleration. Thus it increases linearly with the lateral acceleration and with the square of the speed. In the understeer case, the lateral acceleration at the center of gravity causes the front wheels to slip sideways to a greater extent than at the rear wheels. Thus to develop the lateral force at the front wheels necessary to maintain the radius of turn, the front wheels must be steered to a greater angle.

Oversteer -  $\alpha_f < \alpha_r$ ,  $K < 0$

On a constant-radius turn, the steer angle will have to decrease as the speed (and lateral acceleration) is increased. In this case, the lateral acceleration at the center of gravity causes the slip angle on the rear wheels to increase more than at the front. The outward drift at the rear of the vehicle turns the front wheels inward, thus diminishing the radius of turn. The increase in lateral acceleration that follows causes the rear to drift out even further and the process continues unless the steer angle is reduced to maintain the radius of turn.

## 2 EXPERIMENTAL SETUP

Dynamic testing of the vehicle was performed with the modern measuring equipment on the motor vehicle BMW M3 E36. A sensor used is designed for direct, slip-free measurement of longitudinal and transverse vehicle dynamics which feature high-quality optical elements, the newest optoelectronic components and state-of-the-art high-performance signal processing based on DSP and FPGA's. Speed and distance information is updated at 250 Hz to track every highly dynamic maneuver.

Sensor has several mounting options as shown in Figure 5.

This sensor equipment uses electronics which provide option for connection of a gyro to attain yaw rate for measurement of sideslip angle relative to the vehicle's center of gravity.

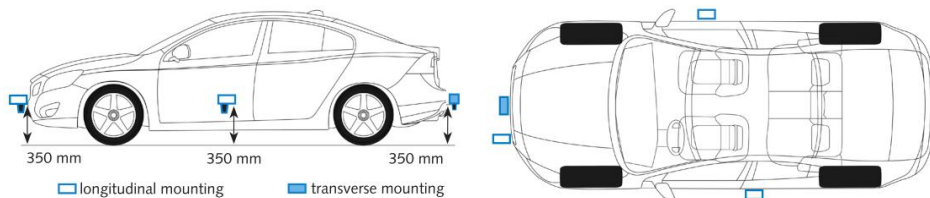


Figure 5. Possible mounting options

In this case, the sensor was mounted on the vehicles door as shown in Figure 6.

Programmable, standardized signal outputs and interfaces allow direct connection to PC and virtually all data acquisition systems, making all measured values directly available.

The testing of the vehicle steerability can be performed in three modes [13]:

- with constant radius ( $R = \text{const.}$ ) while speed changes
- with constant speed ( $V = \text{const.}$ ) while radius changes - preferred method in this research
- with constant steering-wheel angle while speed increases

However, this test was also performed in the so-called "eight" mode, where the vehicle alternately turns left and right, moving along the path in the form of number eight, where the interdependence of driving angle and lateral acceleration was recorded.



Figure 6. Preferred way of mounting during testing

### 3 RESULTS AND ANALYSIS

#### 3.1 Steerability test

The driver gradually turns the steering wheel while maintaining a constant speed. If the vehicle has NEUTRAL steerability, the dependence of yaw angle (including sideslip angle) and the turning radius will be LINEAR. In any other case, it will be curvilinear.

In Figure 7. the changes of vehicle speed and curve radius in relation to time are shown.



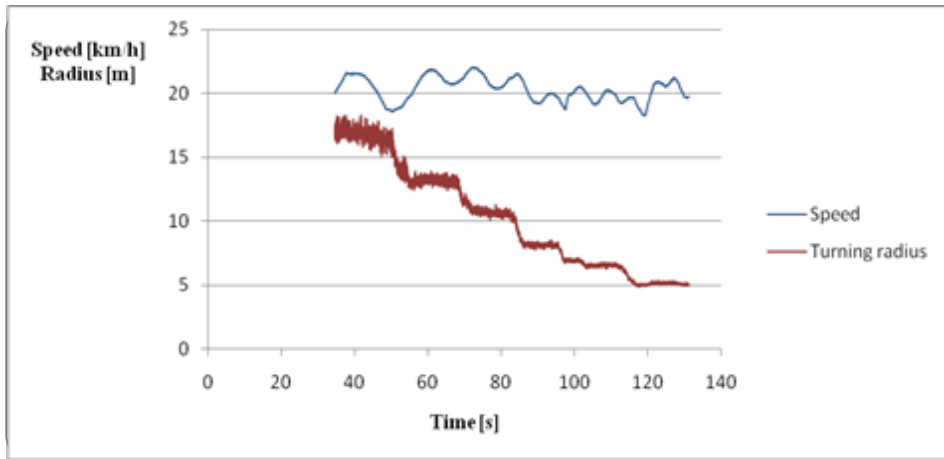


Figure 7. Change of speed and radius in relation to time

Speed was maintained at a value of approximately 20 km/h during the testing. On the other hand, the turning radius was gradually decreased which means that the sideslip angle should be increased.

The obtained results (Figure 8.) depict a linear change of the yaw angle relative to the turning radius, which means that the tested vehicle has neutral steerability.

The dispersion of the measured data (blue dots, Figure 8.) justifies the second test, the "8" test, which goal is to examine the dependence of sideslip angle relative to lateral acceleration (or force).

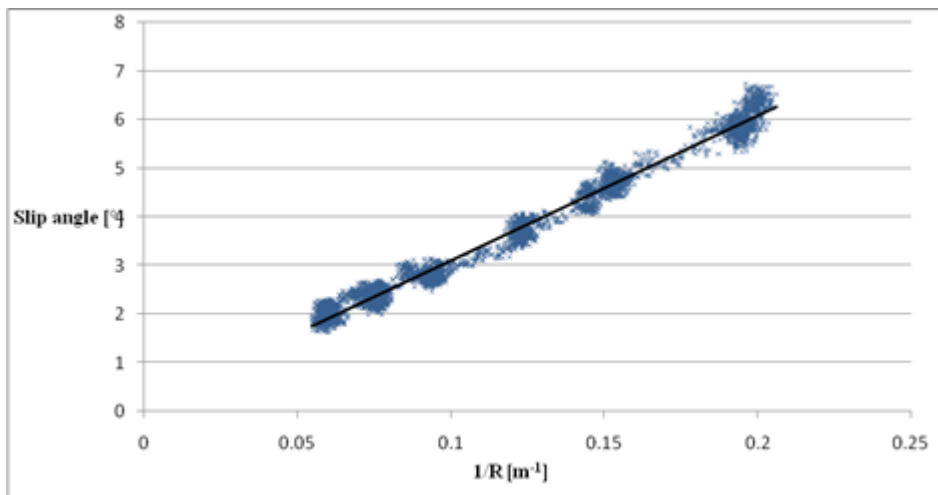


Figure 8. Relation between yaw angle and turning radius

### 3.2 The "8" test

Two recorded cycles were used to analyze the measured sideslip angle during the "8" test. One cycle is shown in Figure 9. The cycles are separated due to the large lateral acceleration difference.

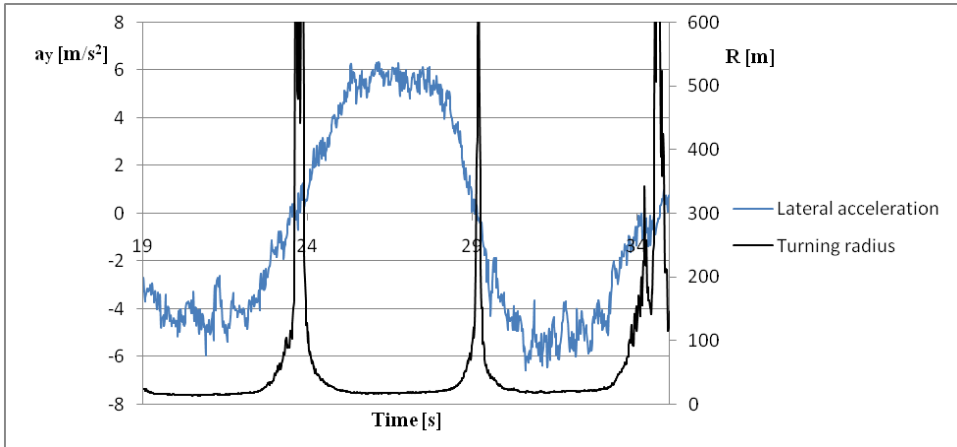


Figure 9. First of the two cycles

Sideslip angle to lateral acceleration dependence is shown in Figures 10 and 11.

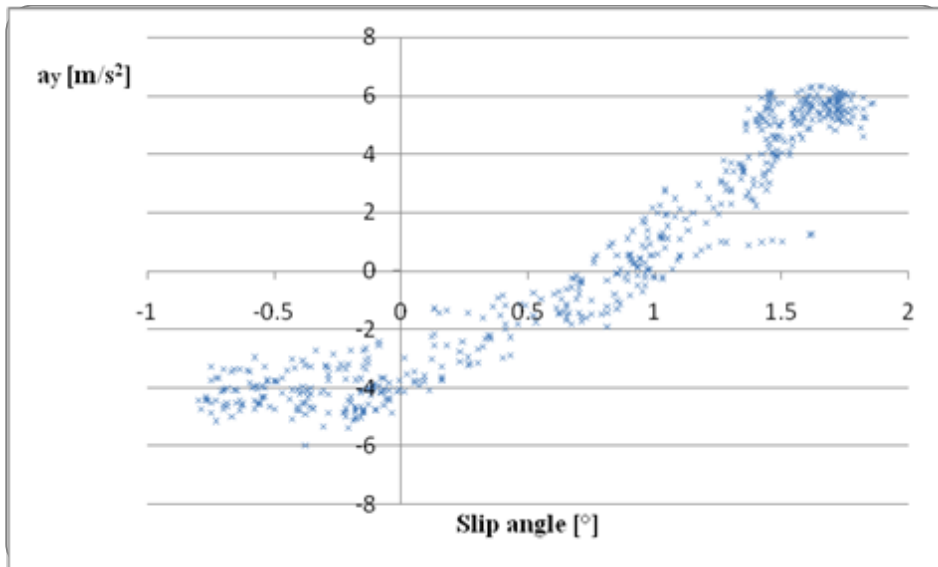


Figure 11. Sideslip angle to lateral acceleration dependence in first cycle

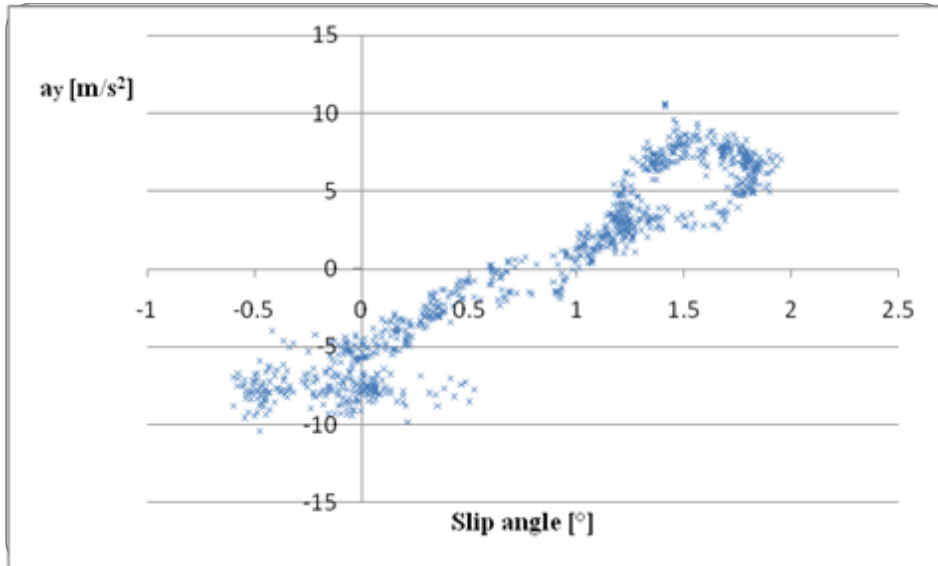


Figure 12. Sideslip angle to lateral acceleration dependence in second cycle

The relationship between these two parameters throughout the entire testing is shown in Figure 13.

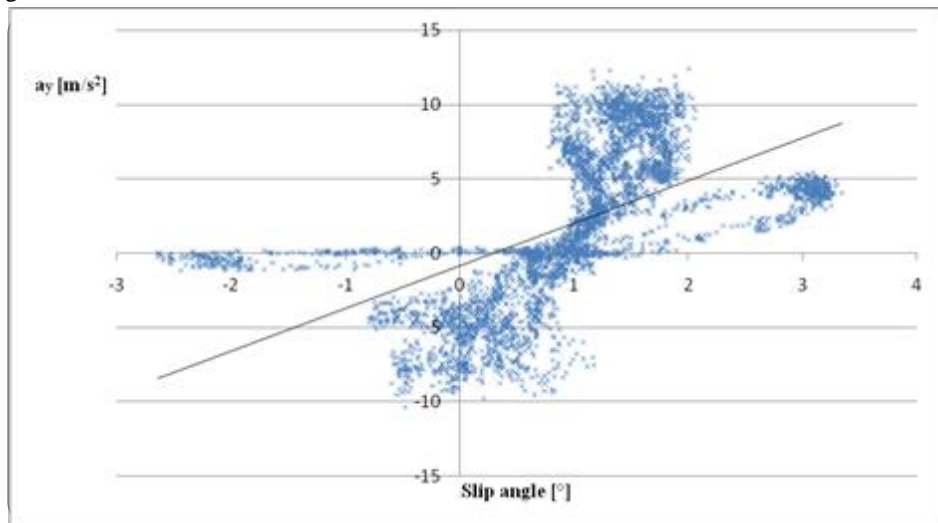


Figure 13. Sideslip angle to lateral acceleration dependence throughout entire testing

Due to not knowing the actual dependence of these two parameters, the linear dependence (straight line, Figure 13) is often adopted in literature. However, in the same figure, it can be seen that the adoption of linear dependence in certain calculations can lead to an error of great margin. Therefore, it is necessary to know other parameters of importance related to the construction of tires, the wheel camber, and the micro and macro structure of the road.

#### 4 CONCLUSION

The aim of this paper was to investigate the influence of sideslip angle on the vehicle steerability and the relative dependence of the sideslip angle and lateral force, which in the literature is most often presented as a linear dependence until the point of lateral sliding, and then as a uniform increase of sideslip angle without further increase of lateral force.

It is shown that modern measuring equipment can be used for fast, simple, and efficient testing of motor vehicle steerability including relatively accurate sideslip angle estimation, however, it is necessary to keep in mind that the obtained results deviate to a certain extent from the real ones due to the great stochasticity of the sideslip angle, ie dependence on numerous other parameters.

It was concluded that a clear relationship can not be formed between the lateral force and the sideslip angle between and that for precise determination of the sideslip angle several parameters need to be known such as tire construction, tire pressure, rim diameter, wheel camber as well as micro and macro road structure.

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